Bayesian Blind Image Deconvolution using a Hyperbolic-Secant prior

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Overview

Plan

1. **Blind Image Deconvolution**
2. The Hyperbolic Secant (HS) distribution
3. Modelling and inference
4. Results
5. Conclusions

Blind Image Deconvolution (BID)

$\mathbf{y}$

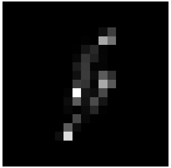
$\overset{?}{\longrightarrow}$

$\mathbf{x}$

Blind Image Deconvolution (BID)

$$\mathbf{y} = \mathbf{h} * \mathbf{x} + \boldsymbol{\eta}, \quad \boldsymbol{\eta} \sim \mathcal{N}\left(\mathbf{0}, \beta^{-1}\mathbf{I}\right)$$

$=$



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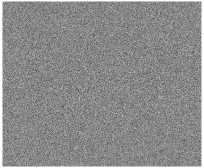

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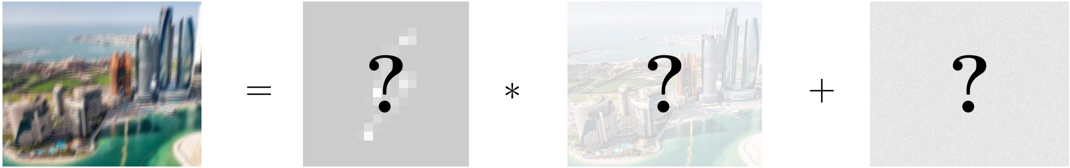

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BID is ill-conditioned

Given $\mathbf{y}$, there are infinitely many $(\mathbf{x}, \mathbf{h})$ such that the above equation holds... we need to restrict the solution space.

Idea: sparsity

When a high pass filter is applied to a sharp image, the resulting image is **sparse**.

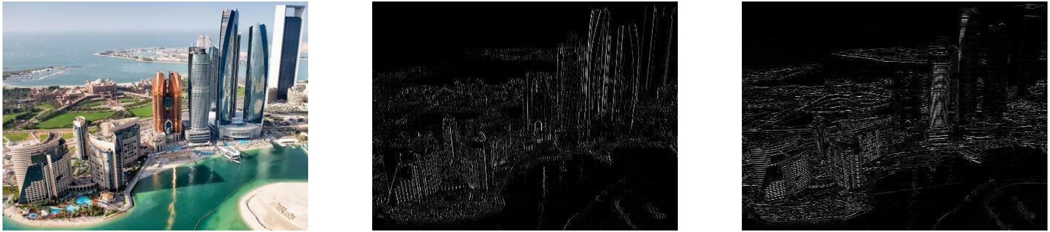

Figure: Clean image and the resulting filtered images.

We need to look for solutions with this property! How? Using Super Gaussian (SG) priors!

Plan

The Hyperbolic Secant (HS) distribution

The Hyperbolic Secant (HS) density is given by

$$f(x;\alpha) = \frac{\alpha}{\pi}\operatorname{sech}(\alpha x) = \frac{2\alpha}{\pi}\left(e^{\alpha x} + e^{-\alpha x}\right)^{-1}, \quad \forall x \in \mathbb{R}.$$

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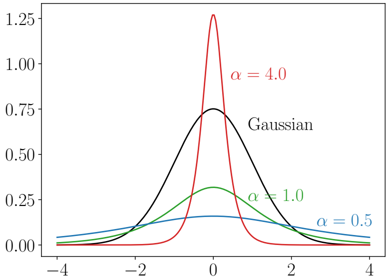

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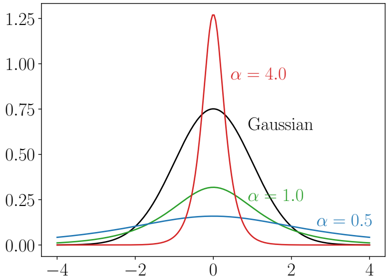

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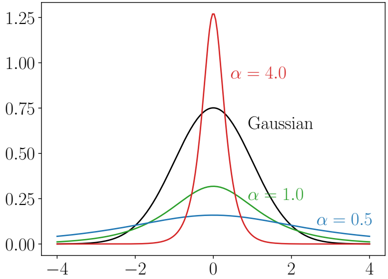

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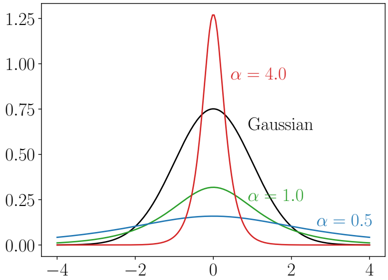

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- Connection to Brownian motion and the Pólya-Gamma distribution.

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Two important properties

1. **Gaussian Scale Mixture (GSM)** representation: there exists a *mixing* density $\widehat{f}(\omega;\alpha)$ such that

$$f(x;\alpha) = \int_0^{+\infty} \mathcal{N}(x \mid 0, \omega^{-1})\widehat{f}(\omega;\alpha)\mathrm{d}\omega.$$

   As a consequence, f is a **Super Gaussian**!

2. f is differentiable around zero (contrary to previously used SGs!)

Plan

Modelling the problem

1. Consider a set of high-pass filters $\{\mathbf{F}_n\}_{n=1}^N$ and apply them to obtain a set of *pseudo-observations*,

$$\underbrace{\mathbf{F}_n \mathbf{y}}_{\mathbf{y}_n} = \mathbf{h} * \underbrace{\mathbf{F}_n \mathbf{x}}_{\mathbf{x}_n} + \underbrace{\mathbf{F}_n \boldsymbol{\eta}}_{\boldsymbol{\eta}_n}.$$

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3. The joint probabilistic model is given by

$$\mathrm{p}\left(\{\mathbf{y}_n\}, \{\mathbf{x}_n\}, \mathbf{h} \mid \boldsymbol{\beta}, \boldsymbol{\alpha}\right) = \mathrm{p}\left(\{\mathbf{y}_n\} \mid \{\mathbf{x}_n\}, \mathbf{h}, \boldsymbol{\beta}\right) \mathrm{p}\left(\{\mathbf{x}_n\} \mid \boldsymbol{\alpha}\right) \mathrm{p}\left(\mathbf{h}\right)$$

Inference

We aim to use mean-field variational inference to approximate

$$\mathrm{p}(\{\mathbf{x}_n\}, \mathbf{h} \mid \{\mathbf{y}_n\}, \boldsymbol{\alpha}, \boldsymbol{\beta}) \approx \mathrm{q}(\{\mathbf{x}_n\})\,\mathrm{q}(\mathbf{h})\,.$$

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Solution: Augmented prior on the filtered images

$$\mathrm{p}\left(\{\mathbf{x}_n\}_{n=1}^{N} \mid \boldsymbol{\omega}\right) \propto \prod_{n=1}^{N}\prod_{i=1}^{HW} \mathcal{N}\left(x_n^i \mid 0, \left(\omega_n^i\right)^{-1}\right), \quad \mathrm{p}(\boldsymbol{\omega} \mid \boldsymbol{\alpha}) = \prod_{n=1}^{N}\prod_{i=1}^{HW} \widehat{f}(\omega_n^i; \alpha_n)$$

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Because of the GSM representation, we recover the original model integrating in $\boldsymbol{\omega}$,

$$\mathrm{p}(\{\mathbf{x}_n\} \mid \boldsymbol{\alpha}) = \int \mathrm{p}(\{\mathbf{x}_n\} \mid \boldsymbol{\omega})\, \mathrm{p}(\boldsymbol{\omega} \mid \boldsymbol{\alpha}) \mathrm{d}\boldsymbol{\omega}$$

Inference in the augmented model

$$\mathrm{p}(\{\mathbf{x}_n\}, \mathbf{h}, \boldsymbol{\omega} \mid \{\mathbf{y}_n\}, \boldsymbol{\alpha}, \boldsymbol{\beta}) \approx \mathrm{q}(\{\mathbf{x}_n\})\, \mathrm{q}(\mathbf{h})\, \mathrm{q}(\boldsymbol{\omega})$$

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- The filtered images are given by $\mathrm{q}(\mathbf{x}_n)=\mathcal{N}\left(\mathbf{x}_n\mid\mathbf{m}_{\mathbf{x}_n},\boldsymbol{\Sigma}_{\mathbf{x}_n}\right)$, with

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- We estimate the blur as the mode of $\mathrm{q}(\mathbf{h})$

$$\widehat{\mathbf{h}} = \arg\min_{\mathbf{h} \in \Delta^K} \left\{ \mathbf{h}^\top \mathbf{C} \mathbf{h} - 2\mathbf{h}^\top \mathbf{b} \right\},$$

$\mathbf{C}$ and $\mathbf{b}$ depend on $\mathbf{m}_{\mathbf{x}_n}$, $\boldsymbol{\Sigma}_{\mathbf{x}_n}$, and $\mathbf{y}_n$.

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$\mathbf{C}$ and $\mathbf{b}$ depend on $\mathbf{m}_{\mathbf{x}_n}$, $\boldsymbol{\Sigma}_{\mathbf{x}_n}$, and $\mathbf{y}_n$.

- For $\mathrm{q}(\boldsymbol{\omega})$ we don't need the full distribution, only its first moment. Its expression is a consequence of the GSM representation,

$$\mathbb{E}_{\mathrm{q}(\omega_n^i)}\left[\omega_n^i\right] = \frac{\alpha_n \tanh(\alpha_n \xi_n^i)}{\xi_n^i}, \quad \xi_n^i = \sqrt{\mathbb{E}_{\mathrm{q}(x_n^i)}\left[(x_n^i)^2\right]}.$$

Algorithm

1. Iterate through the previous updates to approximate the optimal variational distributions,

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2. The mode $\widehat{\mathbf{h}} = \mathrm{argmax}_{\mathbf{h}}\, \mathrm{q}^T(\mathbf{h})$ is used to estimate the latent clean image as

$$\widehat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\{ \frac{1}{2} \left\| \widehat{\mathbf{h}} * \mathbf{x} - \mathbf{y} \right\|^2 + \lambda \sum_{n=1}^{N} \|\mathbf{F}_n \mathbf{x}\|^p \right\},$$

with $p = 0.8$.

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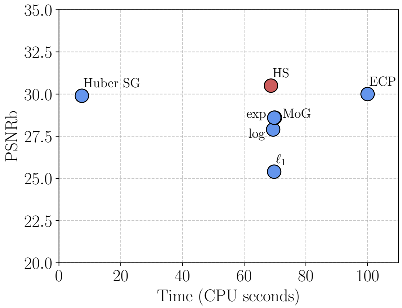
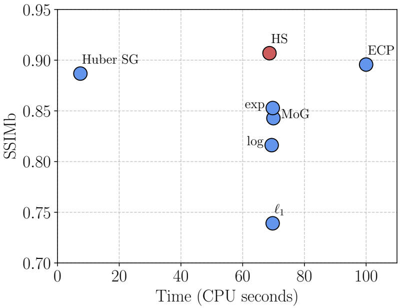
- High-pass filters: first-order horizontal and vertical differences.
- Hyperparameters obtained using grid search: $\beta = 10^4$, $\alpha_1 = 10^{2.4}$, and $\alpha_2 = 10^{2.15}$.

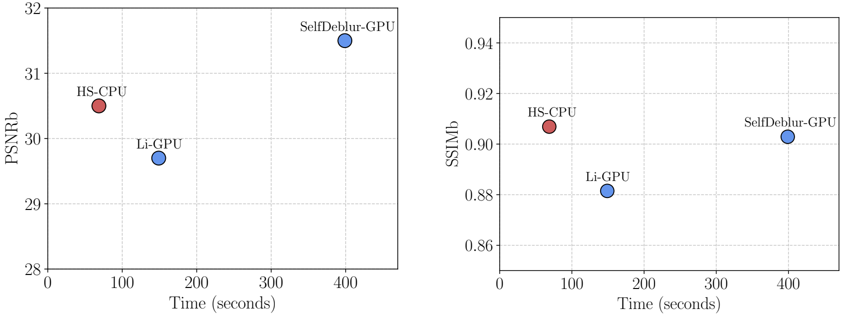
HS vs analytical approaches





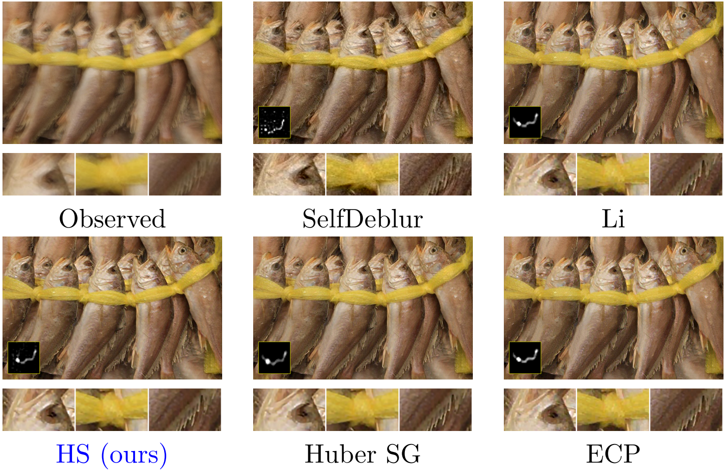
- The proposed HS achieves the best results in terms of absolute performance.
- It is faster than the other methods except Huber SG.

HS vs Deep Learning approaches



- The proposed HS achieves competitive results in terms of absolute performance.
- It is faster than the other methods
- It does not need a GPU.

Comparison using real images



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- The GSM representation provides a new BID Bayesian method.
- Competitive or superior performance in the tested datasets.
- Future work:
  - Automatic hyperparameter estimation.
  - Extensive evaluation on larger datasets.
  - Coupling the HS distribution with Deep Learning methods.

Thank you! ♡